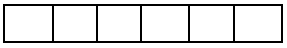
2020 Mathematics Specialist 11: Investigation 1

Take-home part – Syllabus Topic 1.1 (points 1.1.1-1.1.9)

Robot O and Robot X are bored. To pass the time they decide to invent a game using a row with 6 boxes.



They take it in turns to add their own symbol **randomly** to one of the empty spaces on the grid until all the spaces are filled. At the end of each game, there are always three Os and three Xs on the grid e.g.

O	X	X	O	X	O

1. Calculate the total number of distinct completed grids.

In an attempt to add some excitement to the game, Robot O suggests a scoring system, where each row of 3 identical symbols in a completed grid earns 1 point for that player. (Note that the order in which symbols were added is irrelevant; i.e. it doesn't matter which robot created a row of 3 identical symbols first.)

E.g.

O	X	X	X	O	O

earns 1 point for Robot X and 0 points for Robot O.

2. Calculate the following probabilities:
   a) Robot X earns 1 point and Robot O earns 0 points at the end of a game.

   b) Robot X and Robot O each earn 1 point at the end of a game.

   c) Robot X and Robot O each earn 0 points at the end of a game.

Robot O now suggests a new game, to be played on a 3x3 grid. Once again, the robots take turns to add their own symbol **randomly** to one of the empty spaces on the grid. The middle square is left empty so that there are always four Xs and four Os in a completed grid, e.g.

O	X	X
X		O
O	O	X

At the end of each game, points are awarded as before, so that each row of 3 identical symbols (horizontal or vertical) in a completed grid earns 1 point for that player.

E.g.

X	X	O
X		O
O	X	O

scores 1 point for Robot O and 0 points for Robot X.

X	O	O
X		O
X	X	O

scores 1 point for Robot O and 1 point for Robot X.

3. **Investigate** how to calculate probabilities for different outcomes (e.g. the probability that both robots score 0 after a game, or the probability that Robot O scores 1 point, etc.)

In the middle of a thrilling game, the two robots notice that their friend, Robot Y, has arrived.

"Can I join in?" it asks them. "Sure!" reply Robots X and O in unison. They adapt their games so that Robot Y can participate. To help Robot Y learn how to play, they begin with the simpler game, this time using a row of 9 boxes:

The three robots take turns to randomly add their symbol to an empty box, until the grid is complete, e.g.

O	Y	Y	X	O	O	Y	X	X

As before, they use a scoring system where each row of 3 identical symbols in a completed grid earns a point for that robot.

4. **Investigate** how to calculate probabilities for different outcomes (e.g. the probability that all 3 robots score 1 point after a game, or the probability that Robots X and Y both score a point, but Robot O does not).

After a few hundred million games, Robot Y asks "Could we try to play on a 3x3 grid now?"

"Certainly!" says Robot X. "And I guess we should include the middle square now, so that there are 9 squares in total – that way there'll be exactly 3 of each symbol in a completed grid."

"So now there'll be diagonal rows as well – this sounds like fun!"

To be continued…